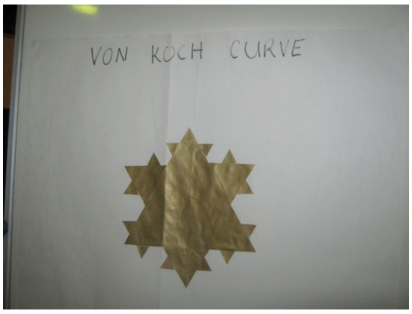
FANTASTIC FRACTALS **The von Koch Curve**

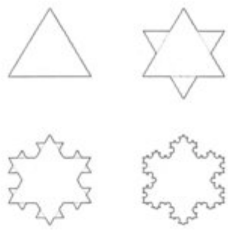
This is also known as the "snowflake curve" – you'll soon see why – but it was first invented by a Swedish mathematician called Helge von Koch in 1904.

A Take a large piece of backing paper, and draw an equilateral triangle with edges of length 27cm.

Step 1 Divide the triangle edge length by 3.
Step 2 Make a triangle with this edge length for each edge of the previous triangle. How many triangles do you need? Record this number each time you go through these instructions.
Step 3 Stick one new triangle to the middle of each of the edges of the previous triangle.
Repeat Steps 1 to 3 until you have made triangles that are 1cm edge length, and stuck them on.



The von Koch curve is the outer edge of the snowflake.
The diagram on the right shows Stages 0, 1, 2 and 3 of the von Koch curve.
The poster above is at Stage 2.
You can imagine the process being repeated again and again and again many tImes although the triangles would actually get too small to handle.

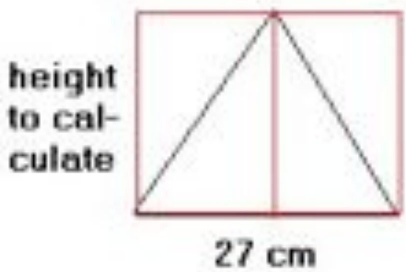


Record the lengths of edges you have made. Can you predict what the lengths of edges of the next few sets of triangles would be? *[It is better to use fractions rather than decimals].*

B The original triangle, with edges of length 27cm, has area 316 cm^2 (can you show that that is correct? See next paragraph if you don't know how to find the area of a triangle yet).

Find the areas of the next 3 sets of triangles (down to the one that has edge length 1cm). As well as doing this by measurement, can you see how to use the information above to get exact figures (these will involve fractions)?

[If you don't know how to find the area of a triangle yet, think about dividing the triangle into two. Now draw lines around each half, so you have two identical rectangles – see the diagram below. You know how to find the area of a rectangle, and you can see how much of the area of the rectangle the triangle occupies. Remember the original triangle has a base length of 27 cm and an area of 316 cm^2 – can you work out the height of the rectangles from that. The next triangles you make will have a base length of 9cm (divide the 27 by 3), so they will have a height which is the previous height divided by 3 also. Remember you need to use fractions rather than decimals for these calculations if you are to be able to see what is happening later on



C Draw up a table for the perimeter in each stage of the von Koch curve like this, adding as many stages as you can:

Stage	No. of triangles added during this step	Length of edge of triangles added	Total no. of edges	Total perimeter of von Koch curve
0	1	27	3	
1	3		12	
2				

- What happens to the length of a single edge at each step?
- What can you say about the number of triangles we would use if we were to keep repeating the basic process? What happens each time you add a step?
- What can you say about the total number of edges? What happens each time you add a step?
- What can you say about the total perimeter if we kept on repeating the process? What happens each time you add a step?

D Now draw up a table for the area in each stage of the von Koch curve like this, adding as many stages as you can (you will probably find it easier to work in decimals for the areas, rather than fractions):

Step	No. of triangles added during this step	Area of triangle added	Total area enclosed by von Koch curve
0	1	316	316
1	3		
2			

- What happens to the area of a triangle at each step?
- What can you say about the total area if we kept on repeating the process?

E How many steps would it take for the perimeter to be greater than the height of the tallest person in the class? How about the perimeter of the classroom? Is the area contained by the von Koch curve ever greater than that of your classroom?

F Try putting a rectangle, or a square, or a hexagon, or a circle around the various stages of the Koch curve. Can you use this information to estimate the greatest area it can contain?